Cheat sheet Tangles, Algebras and Mathematica

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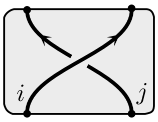
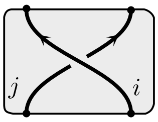
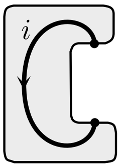
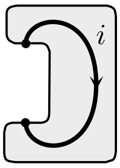
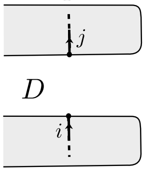
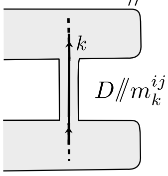
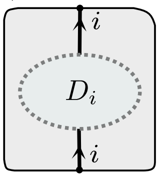
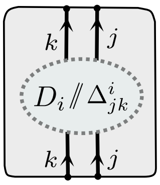
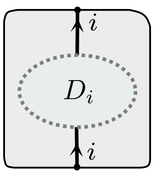
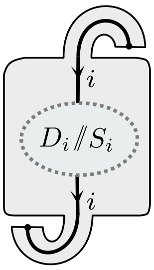
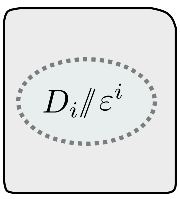
Tangle diagram	Algebra $\mathbb{D}$	Mathematica
Diagram with strands labeled by set L	Element $\mathbb{O}(Pe^G) \in \mathbb{D}^{\otimes L}$	$\mathbb{E}_{\{\}\to L}[L,Q,P]$, $G = L + Q$
 Pos. crossing X_{ij}	R-matrix $R_{ij} \in \mathbb{D}^{\otimes\{i,j\}}$	tR_{ij}
 Neg. crossing $\bar{X}_{ij}$	inverse R-matrix $R_{ij}^{-1} \in \mathbb{D}^{\otimes\{i,j\}}$	$\overline{tR}_{ij}$
 Spinners C_i, $\bar{C}_i$ 	C_i looks like a C. Group-like element $C_i, C_i^{-1} \in \mathbb{D}^{\otimes\{i\}}$ and its inverse.	tC_i $\overline{tC}_i$

Operations on tangles	Linear map $\mathbb{D}^{\otimes J} \to \mathbb{D}^{\otimes K}$	Generating function
Composition is written $/\!\!/$	so $g \circ f = f /\!\!/ g$	$\mathbb{E}_{J\to K}[L,Q,P]$
  Merge, i with j	Algebra multiplication $m_k^{ij} : \mathbb{D}^{\otimes\{i,j\}} \to \mathbb{D}^{\otimes\{k\}}$ the result is named k	$tm_{i,j\to k}$
Disjoint union of diagrams (juxtaposition)	Labeled tensor product $\otimes$, in $\mathbb{D}^{\otimes\{1,2,3\}}$ we often write $x_1 + y_2 z_3$ instead of $x \otimes 1 \otimes 1 + 1 \otimes y \otimes z$	Product
  Double i	co-product $\Delta^i_{j,k} : \mathbb{D}^{\otimes\{i\}} \to \mathbb{D}^{\otimes\{j,k\}}$ the result is placed in tensor factors j, k.	$t\Delta_{i\to j,k}$
  reverse i	Antipode $S_i : \mathbb{D}^{\otimes\{i\}} \to \mathbb{D}^{\otimes\{i\}}$ The inverse antipode S_i^{-1} is similar with hooks the other way.	tS_i $\overline{tS}_i$
 delete i	Co-unit $\varepsilon_i : \mathbb{D}^{\otimes\{i\}} \to \mathbb{Q}$	$\mathbb{E}_{\{i\}\to\{\}}[0,0,1]$

For more see our preprint Perturbed Gaussian generating functions for universal knot invariants
`https://arxiv.org/pdf/2109.02057.pdf`

and the computer implementation at
`www.rolandvdv.nl/PG`